Non-Abelian Strings: From Weak to Strong Coupling and Back via Duality

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Abstract.
    The crossover transition from weak coupling at large ξ to strong coupling at small ξ is studied in $\mathcal{N} = 2$ supersymmetric gauge theory with the U(N) gauge group and $N_f > N$ (here ξ is the Fayet–Iliopoulos parameter). We find that at strong coupling a dual non-Abelian weakly coupled $\mathcal{N} = 2$ theory exists which describes low-energy physics at small ξ. The dual gauge group is U($N_f - N$). The dual theory has N_f flavors of light dyons, to be compared with N_f quarks in the original U(N) theory. Both theories support non-Abelian semilocal strings. In each of these two regimes there are two varieties of physical excitations: elementary fields and nonperturbative composite states bound by confining strings. These varieties interchange upon transition from one regime to the other. We conjecture that the composite stringy states can be related to Seiberg's M fields.

Duality is one the most powerful methods in modern theoretical physics. It allows one to study a theory at strong coupling using its dual formulation which at weak coupling. In this talk new results [1] on non-Abelian duality will be reported.

First, let us ask ourselves what we learned about duality in four-dimensional gauge theories in the last 15 years. The summary is as follows:

(i) the Seiberg–Witten electromagnetic duality in $\mathcal{N} = 2$ SQCD [2, 3] reduces, in the infrared, the underlying non-Abelian theory to an Abelian effective theory. Condensation of monopoles (charges in the dual theory) leads to formation of the Abrikosov–Nielsen–Olesen (ANO) flux tubes (strings). The flux they carry is that of the magnetic field of the dual theory, which is equivalent to the electric flux in the original theory. Thus, they confine quarks. Confinement is essentially Abelian.

(ii) The second example is Seiberg's duality in $\mathcal{N} = 1$ SQCD [4]. It connects the underlying SU(N) theory with N_f flavors with its dual counterpart, having the SU($\tilde{N}$) gauge group ($\tilde{N} = N_f - N$) and N_f flavors of "dual quarks" coupled to a neutral mesonic field M. Seiberg's duality also takes place in the infrared; the ultraviolet behavior of the dual partners is different.

The common belief is that as we deform $\mathcal{N} = 2$ SQCD adding a mass term for the adjoint matter, $\mu \mathrm{Tr}\Phi^2$, the Seiberg–Witten Abelian duality smoothly goes into Seiberg's non-Abelian duality. The monopoles of (i) become "dual quarks" of (ii), and their condensation leads to non-Abelian confinement of quarks.
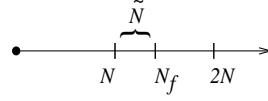
FIGURE 1. The number of flavors exceeds the number of colors, $\tilde{N} \equiv N_f - N > 0$.
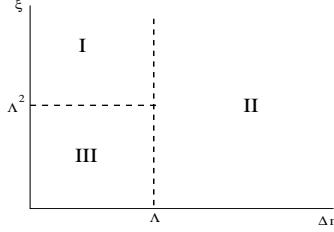

FIGURE 2. Various regimes in $\mathcal{N} = 2$ QCD are separated by crossovers.

The results we report [1] (see also [5]) extend those of [1]; they do *not* fully support the above common belief. We find:

- Duality between two non-Abelian theories. Non-Abelian duality is not the electromagnetic duality. Monopoles are confined in both original and dual theories;
- Both theories in the dual pair have non-Abelian strings which confine;
- We observe small↔large ξ duality in the bulk and on the string world sheet;
- Weakly coupled domain is separated from strongly coupled by a crossover transition with respect to ξ.

Here ξ is the Fayet–Iliopoulos (FI) term, a key element of our set-up which can be summarized as follows. We consider $\mathcal{N} = 2$ SQCD with the gauge group $U(N)$ assuming $N_f > N$ but $N_f < 2N$ to keep asymptotic freedom in the original theory, see Fig. 1. The Fayet–Iliopoulos term $\xi \neq 0$ triggers condensation of N squark fields. The (s)quark mass differences

$$\Delta m_{AB} = m_A - m_B, \qquad A, B = 1, ..., N_f \tag{1}$$

are variable parameters. If ξ is large, the theory is at weak coupling, while at small ξ it becomes strongly coupled. A weakly coupled dual description is constructed at small ξ. Various regimes of the theory in the $\{\xi, \Delta m\}$ plane are schematically shown in Fig. 2.

The bosonic part of our basic theory has the form

$$
\begin{aligned}
S &= \int d^4 x \left[\frac{1}{4g_2^2} \left(F_{\mu\nu}^a \right)^2 + \frac{1}{4g_1^2} \left(F_{\mu\nu} \right)^2 + \frac{1}{g_2^2} \left| D_\mu a^a \right|^2 + \frac{1}{g_1^2} \left| \partial_\mu a \right|^2 \right. \\
&\quad + \left. \left| \nabla_\mu q^A \right|^2 + \left| \nabla_\mu \bar{\tilde{q}}^A \right|^2 + V(q^A, \tilde{q}_A, a^a, a) \right],
\end{aligned}
\tag{2}
$$

where

$$
V(q^A, \tilde{q}_A, a^a, a) = \frac{g_2^2}{2} \left(\frac{1}{g_2^2} f^{abc} \bar{a}^b a^c + \bar{q}_A T^a q^A - \tilde{q}_A T^a \bar{\tilde{q}}^A \right)^2
$$
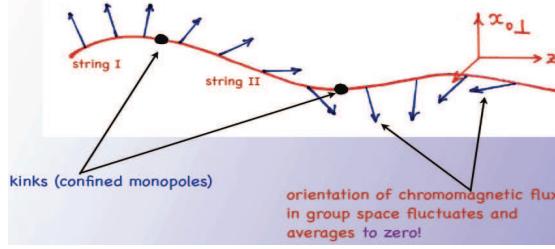
FIGURE 3. Non-Abelian strings.

$$
+\frac{g_1^2}{8}\left(\bar{q}_A q^A - \tilde{q}_A \bar{\tilde{q}}^A - N\xi\right)^2 + 2g_2^2\left|\tilde{q}_A T^a q^A\right|^2 + \frac{g_1^2}{2}\left|\tilde{q}_A q^A\right|^2
$$

$$
+\frac{1}{2}\sum_{A=1}^{N_f}\left\{\left|(a+\sqrt{2}m_A + 2T^a a^a)q^A\right|^2 + \left|(a+\sqrt{2}m_A + 2T^a a^a)\bar{\tilde{q}}^A\right|^2\right\}.
$$

With degenerate quark masses $\Delta m_{AB} = 0$, the microscopic theory has an unbroken global $SU(N)$ symmetry which is a diagonal combination of $SU(N)_{\text{color}}$ and an $SU(N)$ subgroup of the flavor $SU(N_f)$ group acting in the theory. Thus, the color-flavor locking takes place. All light states come in the adjoint and singlet representations of the unbroken $SU(N)_{C+F}$.

The theory supports non-Abelian strings [6, 7] (for reviews see [8]). Since $N_f > N$, these strings are semilocal and would not lead to linear confinement if all Δm's were set to zero. A pictorial representation of non-Abelian strings is given in Fig. 3.

In the original theory the squark fields are condensed in domain I, Fig. 2. The theory is fully Higgsed. The monopoles are attached to strings. In fact, the $SU(N)$ monopoles represent the junctions between two distinct degenerate strings. They are seen as kinks in the world-sheet sigma model [7], see Fig. 3.

The domain II is that of the *Abelian* Higgs regime at weak coupling. As we increase Δm_{AB}, the (off diagonal) W bosons and their superpartners become exceedingly heavier and decouple from the low-energy spectrum. We are left with the photon (diagonal) gauge fields and their quark $\mathcal{N} = 2$ superpartners. Explicit breaking of the flavor symmetry by $\Delta m_{AB} \neq 0$ leads to the loss of the non-Abelian nature of the string solutions; they become Abelian (the so-called Z_N) strings.

Finally, as we reduce ξ and $|\Delta m_{AB}|$ below Λ, we enter the strong coupling domain III. We show that at $N_f > N$ there is a dual description in this domain, moreover, the dual theory is non-Abelian, with the dual gauge group

$$
U(\tilde{N}) \times U(1)^{N-\tilde{N}}, \tag{3}
$$

and N_f flavors of charged non-Abelian dyons. In its gross features the dual $\mathcal{N} = 2$ theory that we found is similar to Seiberg's dual [4] to our original microscopic theory. Because $N_f > 2\tilde{N}$, the dual theory is infrared (IR) free rather than asymptotically free. This result is in perfect match with the results obtained in [9] where the dual non-Abelian gauge group $SU(\tilde{N})$ was identified at the root of a baryonic branch in the $SU(N)$ gauge theory with massless quarks (see also [10]).

In the limit of degenerate quark masses $\Delta m_{AB} = 0$ and small ξ, the dual theory has an unbroken global diagonal $SU(\tilde{N})$ symmetry. It is obtained as a result of the spontaneous breaking of the gauge $U(\tilde{N})$ group and an $SU(\tilde{N})$ subgroup of the flavor $SU(N_f)$ group. Thus, the color-flavor locking takes place in the dual theory as well, much in the same way as in the original microscopic theory in the domain I, albeit the preserved diagonal symmetry is different. The light states come in adjoint and singlet representations of the global $SU(\tilde{N})_{C+F}$. Thus, the low-energy spectrum of the theory in the domain III is dramatically different from that of domain I. Excitation spectra are arranged in different representations of the global unbroken groups, $SU(N)$ and $SU(\tilde{N})$, respectively.

Three above-mentioned regimes — three domains shown in Fig. 2 — are arguably separated by crossovers, much in the same way as it happens in the case $N_f = N$ [5]. The evidence in favor of crossovers (rather than phase transitions) can be summarized as follows.

- In the equal quark mass limit the domains I and III have Higgs branches of the same dimensions and the same pattern of global symmetry breaking;
- For generic masses $\Delta m_{AB} \neq 0$ all three regimes have the same number of isolated vacua at nonvanishing ξ;
- Each of these vacua has the same number ($= N$) of distinct elementary strings in all three domains. Moreover, the BPS spectra of excitations on the non-Abelian string coincide in the domains I and III.

BASIC FEATURES OF THE SET-UP

The field content is as follows. The $\mathcal{N} = 2$ vector multiplet consists of the U(1) gauge field A_μ and the $SU(N)$ gauge field A_μ^a, where $a = 1, ..., N^2 - 1$, and their Weyl fermion superpartners plus complex scalar fields a, and a^a and their Weyl superpartners. The N_f quark multiplets of the $U(N)$ theory consist of the complex scalar fields q^{kA} and $\tilde{q}_{Ak}$ (squarks) and their fermion superpartners, all in the fundamental representation of the $SU(N)$ gauge group. Here $k = 1, ..., N$ is the color index while A is the flavor index, $A = 1, ..., N_f$. We will treat q^{kA} and $\tilde{q}_{Ak}$ as rectangular matrices with N rows and N_f columns.

Let us discuss the vacuum structure of this theory. The vacua of the theory (2) are determined by the zeros of the potential V. With the generic choice of the quark masses we have $C_{N_f}^N = N_f!/N!\tilde{N}!$ isolated r-vacua in which $r = N$ quarks (out of N_f) develop vacuum expectation values (VEVs). Consider, say, the $(1,2,...,N)$ vacuum in which the first N flavors develop VEVs. We can exploit gauge rotations to make all squark VEVs real. Then in the problem at hand they take the form

$$\langle q^{kA}\rangle = \sqrt{\xi} \begin{pmatrix} 1 & ... & 0 & 0 & ... & 0 \\ ... & ... & ... & ... & ... & ... \\ 0 & ... & 1 & 0 & ... & 0 \end{pmatrix}, \qquad \langle \bar{\tilde{q}}^{kA}\rangle = 0, \qquad (4)$$

where we write down the quark fields as matrices in color and flavor indices ($k = 1, ..., N$, $A = 1, ..., N_f$). The FI term ξ singles out the $r = N$ vacua from the set of all r-vacua.

In the vacuum under consideration the adjoint fields also develop VEVs, namely,

$$\left\langle\left(\frac{1}{2}a + T^a a^a\right)\right\rangle = -\frac{1}{\sqrt{2}}\begin{pmatrix} m_1 & \dots & 0 \\ \dots & \dots & \dots \\ 0 & \dots & m_N \end{pmatrix}, \tag{5}$$

For generic values of the quark masses, the SU(N) subgroup of the gauge group is broken down to U(1)$^{N-1}$. However, in the special limit

$$m_1 = m_2 = \dots = m_{N_f}, \tag{6}$$

the SU(N)×U(1) gauge group remains unbroken by the adjoint field. In this limit the theory acquires a global flavor SU(N_f) symmetry.

While the adjoint VEVs do not break the SU(N)×U(1) gauge group in the limit (6), the quark condensate (4) results in the spontaneous breaking of both gauge and flavor symmetries. A diagonal global SU(N) combining the gauge SU(N) and an SU(N) subgroup of the flavor SU(N_f) group survives, however. We refer to this diagonal global symmetry as to SU(N)$_{C+F}$. The color-flavor locking takes place in a slightly different way than in the case $N_f = N$ (or $\tilde{N} = 0$). The presence of the global SU(N)$_{C+F}$ group is instrumental for formation of the non-Abelian strings.

More exactly, the pattern of breaking of the color and flavor symmetry is as follows:

$$\mathrm{U}(N)_{\text{gauge}} \times \mathrm{SU}(N_f)_{\text{flavor}} \to \mathrm{SU}(N)_{C+F} \times \mathrm{SU}(\tilde{N})_F \times \mathrm{U}(1), \tag{7}$$

where $\tilde{N} = N_f - N$. Here SU($\tilde{N}$)$_F$ factor stands for the flavor rotation of the $\tilde{N}$ quarks. For unequal quark masses the global symmetry (7) is broken down to U(1)$^{N_f-1}$.

All gauge bosons in the bulk are massive,

$$m_\gamma = g_1 \sqrt{\frac{N}{2}\xi}, \qquad m_W = g_2 \sqrt{\xi}. \tag{8}$$

The adjoint fields a and a^a as well as N^2 components of the quark matrix q acquire the same masses as the corresponding gauge bosons.

A JOURNEY THROUGH VARIOUS REGIMES

We will single out a group of N quarks and another one of $\tilde{N}$ quarks. We generically refer to the masses in the first and second groups as m_P and m_K, respectively; $P = 1, \dots, N$ numerates the quark flavors which develop VEVs, while $K = N+1, \dots, N_f$ numerates "extra" quark flavors. The extra flavors become massless in the limit (6). The mass differences inside the first group (or inside the second group) are called ΔM_{inside}. The mass differences $m_P - m_K$ are referred to as $\Delta M_{\text{outside}}$. The transitions we will study are those in ξ (the vertical axis) and ΔM_{inside} (the horizontal axis in Fig. 4). At $\xi = 0$ we arrive at the Seiberg–Witten solution. Note that $m_P - m_K \equiv \Delta M_{\text{outside}}$ is kept fixed in the process. Eventually, we take $\Delta M_{\text{outside}} \ll \Lambda$. This is necessary to get the dual theory non-Abelian.
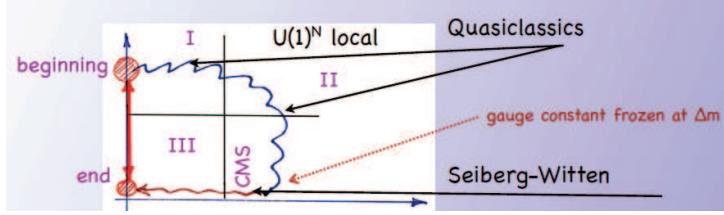
FIGURE 4. The transitions in the $\{\xi, \Delta M_{\text{inside}}\}$ plane.
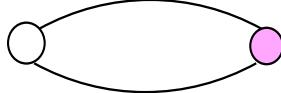
FIGURE 5. Meson formed by antimonopole and dyon connected by two strings. Open and closed circles denote dyon and antimonopole, respectively.

If $m_P - m_K \equiv \Delta M_{\text{outside}} \neq 0$, the extra quark flavors acquire masses determined by the mass differences $m_P - m_K$.

Note that all states come in representations of the unbroken global group (7), namely, the singlet and adjoint representations of $SU(N)_{C+F}$

$$(1, 1), \quad (N^2 - 1, 1), \tag{9}$$

and bifundamentals

$$(\bar{N}, \tilde{N}), \quad (N, \bar{\tilde{N}}), \tag{10}$$

where we mark representation with respect to two non-Abelian factors in (7).

In the beginning we have the gauge group $U(N)$, with N_f matter hypermultiplets. The light part of the spectrum includes the vector supermultiplet ($16 \times (N^2 - 1)$ degrees of freedom with mass $\sim g\sqrt{\xi}$) plus extra bifundamentals (10). In addition, there are $\tilde{N}^2 - 1$ composites of the type presented in Fig. 5. Their mass is heavy (i.e. $\sim \sqrt{\xi}$). In the end we have the gauge group $U(\tilde{N})$, with $\tilde{N}$ matter hypermultiplets. The light part of the spectrum includes the vector supermultiplet ($16 \times (\tilde{N}^2 - 1)$ degrees of freedom with mass $\sim g\sqrt{\xi}$). In addition, there are extra bifundamentals (10) and $N^2 - 1$ heavy composites $D\bar{D}$.

NON-ABELIAN STRINGS

The Z_N-string solutions in the theory with $N_f = N$ break the $SU(N)_{C+F}$ global group. Therefore, strings have orientational zero modes, associated with rotations of their color flux inside the non-Abelian $SU(N)$. The global group is broken down to $SU(N-1) \times U(1)$. As a result, the moduli space of the non-Abelian string is described by the coset space

$$\frac{SU(N)}{SU(N-1) \times U(1)} \sim CP(N-1). \tag{11}$$

The low-energy effective theory on the world sheet of the non-Abelian string is $\mathcal{N} = 2$ SUSY two-dimensional $CP(N-1)$ model [6, 7].

Now we add $\tilde{N}$ "extra" quark flavors (first, with degenerate masses). Then the strings become semilocal, whose transverse size is a modulus. In this case these strings do not generate linear confinement. However, at the end, $\Delta M_{\text{outside}} \neq 0$ will lift the size moduli, so that linear confinement will ensue.

Non-Abelian semilocal strings have two types of moduli: orientational and size moduli. The orientational zero modes are parametrized by a complex vector n^P, $P = 1, ..., N$, while its $\tilde{N} = (N_f - N)$ size moduli are parametrized by another complex vector ρ^K, $K = N+1, ..., N_f$. The effective two-dimensional theory which describes internal dynamics of the non-Abelian semilocal string is an $\mathcal{N} = (2, 2)$ "toric" sigma model including both types of fields. Its bosonic action in the gauge formulation (which assumes taking the limit $e^2 \to \infty$) has the form

$$S = \int d^2x \left\{ \left| \nabla_\alpha n^P \right|^2 + \left| \tilde{\nabla}_\alpha \rho^K \right|^2 + \frac{1}{4e^2} F_{\alpha\beta}^2 + \frac{1}{e^2} |\partial_\alpha \sigma|^2 \right.$$

$$+ \left. 2 \left| \sigma + \frac{m_P}{\sqrt{2}} \right|^2 |n^P|^2 + 2 \left| \sigma + \frac{m_K}{\sqrt{2}} \right|^2 |\rho^K|^2 + \frac{e^2}{2} \left(|n^P|^2 - |\rho^K|^2 - 2\beta \right)^2 \right\},$$

$$P = 1, ..., N, \qquad K = N+1, ..., N_f, \qquad \tilde{\nabla}_k = \partial_k + iA_k. \qquad (12)$$

The fields n^P and ρ^K have charges +1 and −1 with respect to the auxiliary U(1) gauge field, hence, the difference in the covariant derivatives, $\nabla_i = \partial_i - iA_i$ and $\tilde{\nabla}_j = \partial_j + iA_j$ respectively.

The D-term condition

$$|n^P|^2 - |\rho^K|^2 = 2\beta, \qquad (13)$$

is implemented in the limit $e^2 \to \infty$. Moreover, in this limit the gauge field A_α and its $\mathcal{N} = 2$ bosonic superpartner σ become auxiliary and can be eliminated. The two-dimensional coupling constant β is related to the four-dimensional one as $\beta = 2\pi/g_2^2$.

MONODROMIES, OR WHAT BECOMES OF THE QUARK FIELDS IN THE JOURNEY

To simplify presentation, we will consider a particular example, $N = 3$ and $\tilde{N} = 2$. In analyzing the transition from domain I to III (see Fig. 2) we make two steps. First, we take the quark mass differences to be large, passing to domain II. In this domain the theory stays at weak coupling, and we can safely decrease the value of the FI parameter ξ. Next, we use the exact Seiberg–Witten solution of the theory on the Coulomb branch [2, 3] (i.e. at $\xi = 0$) to perform the passage from domain II to III.

In this journey we will have to carefully consider two Argyres–Douglas points, to deal with two monodromies, as we vary m_3, see Fig. 6. In each passage we split the masses, first m_1 and m_4, and then m_2 and m_5. At the end we tend $m_1 \to m_2 \to m_3$ and $m_5 \to m_4$
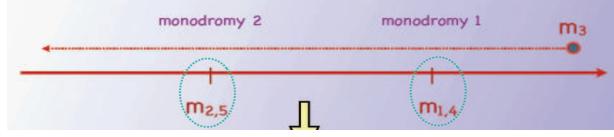
FIGURE 6. Two Argyres–Douglas points on the way from domain I to III.

We investigate the monodromies using the approach of Ref. [5] which is similar to that of Ref. [10]. We start with the $r = 3$ vacuum at large ξ in domain I, where three quarks with charges

$$\left(n_e, n_m; n_e^3, n_m^3; n_e^8, n_m^8\right) = \left(\frac{1}{2}, 0; \frac{1}{2}, 0; \frac{1}{2\sqrt{3}}, 0\right),$$

$$\left(n_e, n_m; n_e^3, n_m^3; n_e^8, n_m^8\right) = \left(\frac{1}{2}, 0; -\frac{1}{2}, 0; \frac{1}{2\sqrt{3}}, 0\right),$$

$$\left(n_e, n_m; n_e^3, n_m^3; n_e^8, n_m^8\right) = \left(\frac{1}{2}, 0; 0, 0; -\frac{1}{\sqrt{3}}, 0\right), \tag{14}$$

develop VEV's. Here n_e and n_m denote electric and magnetic charges of a given state with respect to the U(1) gauge group, while n_e^3, n_m^3 and n_e^8, n_e^8 stand for the electric and magnetic charges with respect to the Cartan generators of the SU(3) gauge group (broken down to U(1)×U(1) by Δm_{AB}).

Then in domain III these quarks transform into light dyons with charges

$$D^{11}: \quad \left(\frac{1}{2}, 0; \frac{1}{2}, \frac{1}{2}; \frac{1}{2\sqrt{3}}, \frac{\sqrt{3}}{2}\right),$$

$$D^{22}: \quad \left(\frac{1}{2}, 0; -\frac{1}{2}, -\frac{1}{2}; \frac{1}{2\sqrt{3}}, \frac{\sqrt{3}}{2}\right),$$

$$D_{33}: \quad \left(\frac{1}{2}, 0; 0, 0; -\frac{1}{\sqrt{3}}, -\sqrt{3}\right). \tag{15}$$

For consistency of our analysis it is instructive to consider another route from the domain I to the domain III, namely the one along the line $\Delta M_{\text{inside}} = 0$. On this line we keep the global color-flavor locked group unbroken. Then we obtain a surprising result: the quarks and gauge bosons which form the adjoint $(N^2 - 1)$ representation of SU(N) at large ξ and the dyons and gauge bosons which form the adjoint $(\tilde{N}^2 - 1)$ representation of SU($\tilde{N}$) at small ξ are, in fact, *distinct* states. How can this occur?

Since we have a crossover between the domains I and III rather than a phase transition, this means that in the full microscopic theory the $(N^2 - 1)$ adjoints of SU(N) become heavy and decouple as we pass from the domain I to III along the line $\Delta m_{AB} = 0$. Moreover, some composite $(\tilde{N}^2 - 1)$ adjoints of SU($\tilde{N}$), which are heavy and invisible in the low-energy description in the domain I become light in the domain III and form the
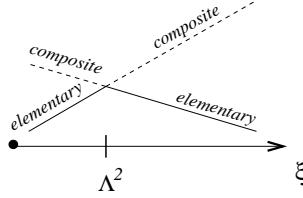
FIGURE 7. Evolution of the SU(N) and SU($\tilde{N}$) W bosons vs. ξ.

D^{lK} dyons ($K = N+1, ..., N_f$) and gauge bosons B_μ^p. The phenomenon of level crossing takes place. Although this crossover is smooth in the full theory, from the standpoint of the low-energy description the passage from the domain I to III means a dramatic change: the low-energy theories in these domains are completely different; in particular, the degrees of freedom in these theories are different.

This logic leads us to the following conclusion. In addition to light dyons and gauge bosons included in the low-energy theory in the domain III at small ξ, we have heavy fields (with masses of the order of Λ) which form the adjoint representation $(N^2 - 1, 1)$ of the global symmetry. These are screened (former) quarks and gauge bosons from the domain I continued into III. Let us denote them as $M_P^{P'}$ ($P, P' = 1, ..., N$). What is their physical nature in the region III?

Before answering this question let us note that by the same token, it is seen that in domain I, in addition to the light quarks and gauge bosons, we have heavy fields $M_K^{K'}$ ($K, K' = N+1, ..., N_f$), which form the adjoint ($\tilde{N}^2 - 1$) representation of SU($\tilde{N}$). This is schematically depicted in Fig. 7.

Now we come back to the physical nature of adjoint fields $M_P^{P'}$ in the region III. It is well known that the W bosons usually do not exist as localized states in the strong coupling regime on the Coulomb branch (speaking in jargon, they "decay"). They split into antimonopoles and dyons on CMS on which the Argyres–Douglas points lie [2, 11].

Consider, for example, the W-boson associated with the T^3 generator (T^3 W boson for short) with the charge $(0, 0; 1, 0; 0, 0)$ in the domain II. As we go through CMS this W boson decay into the T^3 antimonopole and dyon with the charges $(0, 0; 0, -1; 0, 0)$ and $(0, 0; 1, 1; 0, 0)$, respectively. It means that the W boson is absent in domain III, in full accord with the analysis of the SU(2) theory in [11].

This picture is valid on the Coulomb branch at $\xi = 0$. As we switch on small $\xi \neq 0$ the monopoles and dyons become confined by strings. In fact, the elementary monopoles/dyons are represented by junctions of two different elementary non-Abelian strings [12, 7], see also a detailed discussion of the monopole/dyon confinement in [1]. This means that, as we move from the domain II into III at small nonvanishing ξ the W boson "decays" into an antimonopole and dyon; however, these states cannot abandon each other and move far apart because they are confined. Therefore, the W boson evolves into a stringy meson formed by an antimonopole and dyon connected by two strings, as shown in Fig. 5, see [8] for a discussion of these stringy mesons.

These stringy mesons have nonvanishing U(1) global charges with respect to the Cartan generators of the SU(3) subgroup of the global group (7) (above we discussed only one W boson of this type, related to the T^3 generator, however, in fact, we have

six different charged gauge boson/quark states of this type). In the equal mass limit these globally charged stringy mesons combine with neutral (with respect to the group $U(1)^{N_f-1}$ stringy mesons formed by pairs of monopoles and antimonopoles (or dyons and antidyons) connected by two strings, to form the octet representation of the SU(3) subgroup of the global group (7) (in general, the adjoint representation of SU(N)). They are heavy in the domain III, with mass of the order of Λ.

We identify these stringy mesons with $(N^2 - 1)$ adjoints $M_P^{P'}$ $(P, P' = 1, ..., N)$ of the SU(N) subgroup with which we have seen *en route* from the domain I to III along the line $\Delta m_{AB} = 0$.

The same applies to the q^{kK} quarks $(K = N+1, ..., N_f)$ of the domains I and II. As we go through the crossover into the domain III at small ξ q^{kK} quarks evolve into stringy mesons formed by pairs of antimonopoles and dyons connected by two strings, see Fig. 5. However, these states are unstable. To see that this is indeed the case, please, observe that in the equal mass limit these stringy mesons fill the bifundamental representations $(N, \bar{\tilde{N}})$ and $(\bar{N}, \tilde{N})$ of the global group; hence, can decay into light dyons/dual gauge bosons with the same quantum numbers.

It is quite plausible to suggest that these fields $M_P^{P'}$ and $M_K^{K'}$ are Seiberg's mesonic fields [4, 13], which occur in the dual theory upon breaking of $\mathcal{N} = 2$ supersymmetry by the mass-term superpotential $\mu \mathscr{A}^2$ for the adjoint fields when we take the limit $\mu \to \infty$. In this limit our theory becomes $\mathcal{N} = 1$ SQCD. Previously, these M_{AB} fields were not identified in the $\mathcal{N} = 2$ theory.

In conclusion, we demonstrated that non-Abelian confinement in our theory is a combined effect of the Higgs screening, "decay" processes on CMS and confining string formation. The strings that are dynamically formed always confine monopoles or dyons (whose charges can be represented as a sum of those of a monopole plus W-boson) both, in the original and dual theories, rather than quarks.

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